Edexcel Maths M2

Topic Questions from Papers

Centre of Mass

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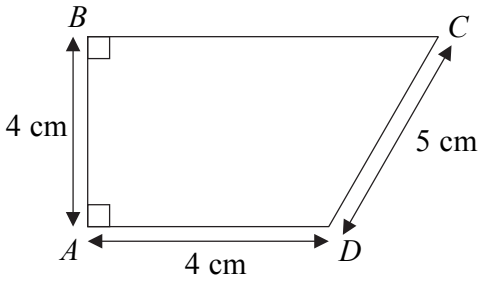
2. **Figure 1**



A thin uniform wire, of total length 20 cm, is bent to form a frame. The frame is in the shape of a trapezium $ABCD$, where $AB = AD = 4$ cm, $CD = 5$ cm, and AB is perpendicular to BC and AD, as shown in Figure 1.

(a) Find the distance of the centre of mass of the frame from AB.

(5)

The frame has mass M. A particle of mass kM is attached to the frame at C. When the frame is freely suspended from the mid-point of BC, the frame hangs in equilibrium with BC horizontal.

(b) Find the value of k.

(3)

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4. **Figure 1**

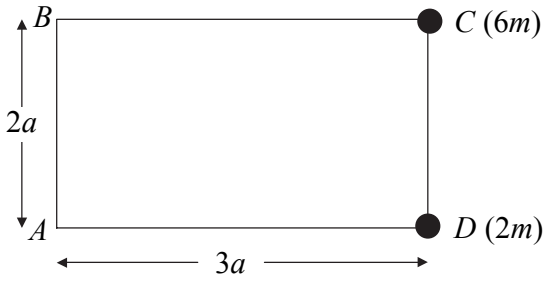

Figure 1 shows four uniform rods joined to form a rigid rectangular framework $ABCD$, where $AB = CD = 2a$, and $BC = AD = 3a$. Each rod has mass m. Particles, of mass $6m$ and $2m$, are attached to the framework at points C and D respectively.

(a) Find the distance of the centre of mass of the loaded framework from

(i) AB,

(ii) AD.

(7)

The loaded framework is freely suspended from B and hangs in equilibrium.

(b) Find the angle which BC makes with the vertical.

(3)

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Question 4 continued

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Q4

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3. **Figure 1**

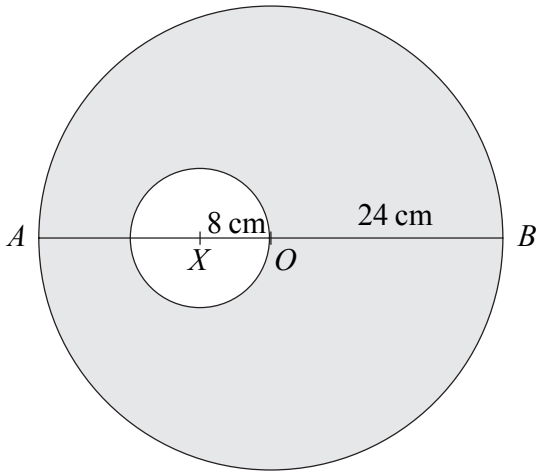

Figure 1 shows a template T made by removing a circular disc, of centre X and radius 8 cm, from a uniform circular lamina, of centre O and radius 24 cm. The point X lies on the diameter AOB of the lamina and $AX = 16$ cm. The centre of mass of T is at the point G.

(a) Find AG.

(6)

The template T is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of T, which passes through the mid-point of OB. A small stud of mass $\frac{1}{4}m$ is fixed at B, and T and the stud are in equilibrium with AB horizontal. Modelling the stud as a particle,

(b) find the mass of T in terms of m.

(4)

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Question 3 continued

Q3

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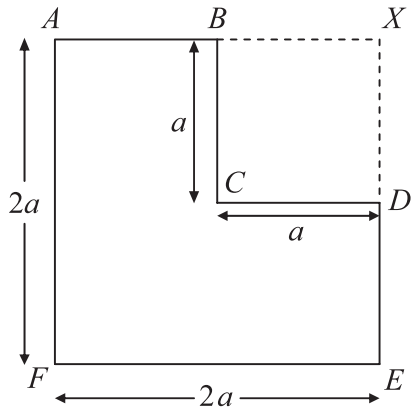

Figure 1

A uniform lamina $ABCDEF$ is formed by taking a uniform sheet of card in the form of a square $AXEF$, of side $2a$, and removing the square $BXDC$ of side a, where B and D are the mid-points of AX and XE respectively, as shown in Figure 1.

(a) Find the distance of the centre of mass of the lamina from AF.

(4)

The lamina is freely suspended from A and hangs in equilibrium.

(b) Find, in degrees to one decimal place, the angle which AF makes with the vertical.

(4)

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Question 3 continued

Q3

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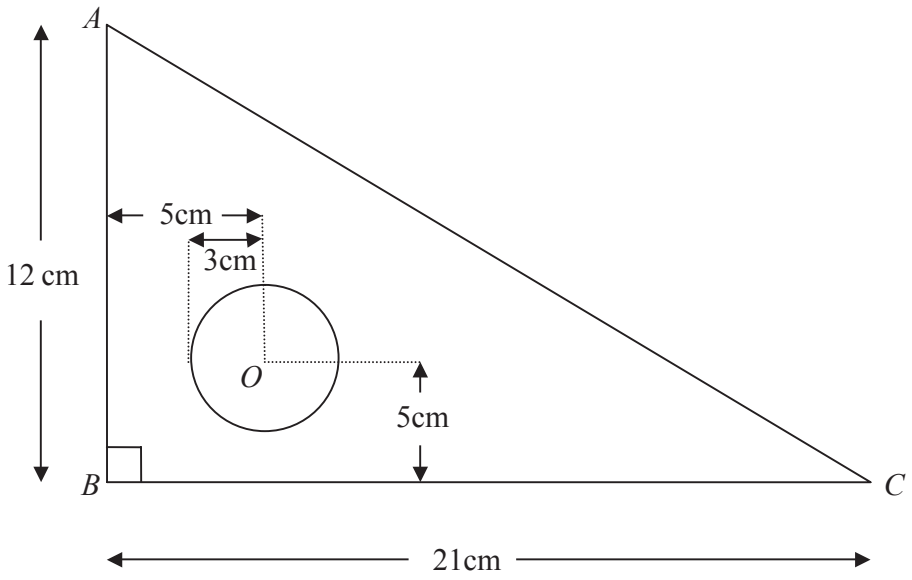

Figure 1

A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC, with $\angle ABC = 90°$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC, as shown in Figure 1.

(a) Find the distance of the centre of mass of S from

(i) AB,

(ii) BC.

(9)

The set square is freely suspended from C and hangs in equilibrium.

(b) Find, to the nearest degree, the angle between CB and the vertical.

(3)

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Question 4 continued

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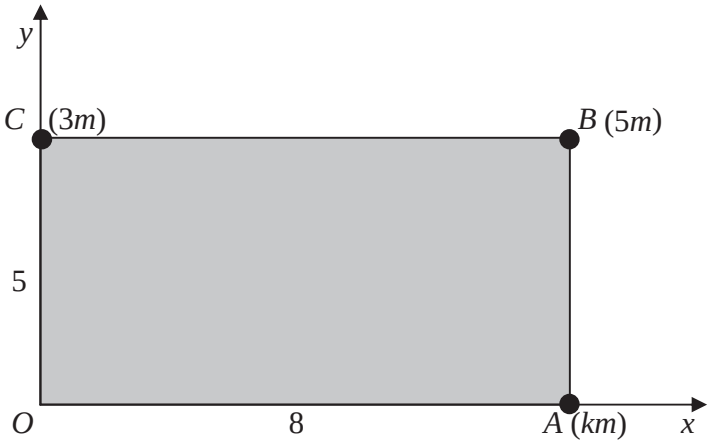

Figure 3

Figure 3 shows a rectangular lamina $OABC$. The coordinates of O, A, B and C are (0, 0), (8, 0), (8, 5) and (0, 5) respectively. Particles of mass km, $5m$ and $3m$ are attached to the lamina at A, B and C respectively.

The x-coordinate of the centre of mass of the three particles *without the lamina* is 6.4.

(a) Show that $k = 7$.

(4)

The lamina $OABC$ is uniform and has mass $12m$.

(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina.

(6)

The combined system is freely suspended from O and hangs at rest.

(c) Find the angle between OC and the horizontal.

(3)

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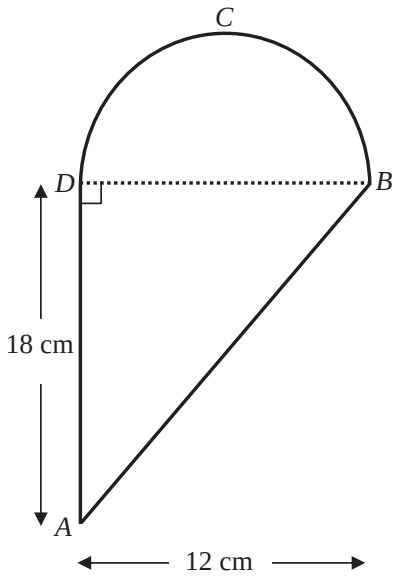

Figure 2

A uniform lamina *ABCD* is made by joining a uniform triangular lamina *ABD* to a uniform semi-circular lamina *DBC*, of the same material, along the edge *BD*, as shown in Figure 2. Triangle *ABD* is right-angled at *D* and *AD* = 18 cm. The semi-circle has diameter *BD* and *BD* = 12 cm.

(a) Show that, to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *AD* is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius *r*, is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

(b) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *BD*.

(4)

The lamina is freely suspended from *B* and hangs in equilibrium.

(c) Find, to the nearest degree, the angle which *BD* makes with the vertical.

(4)

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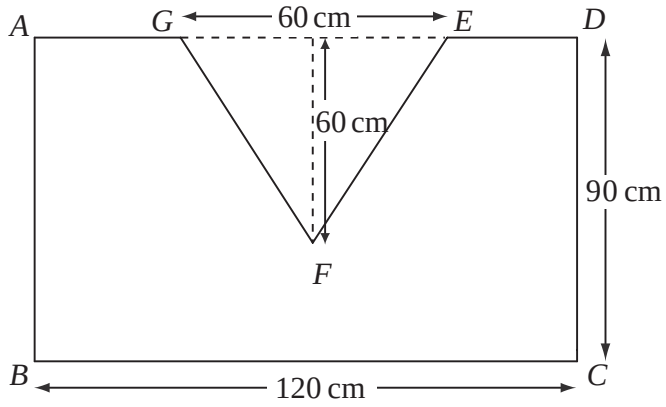

Figure 2

A shop sign *ABCDEFG* is modelled as a uniform lamina, as illustrated in Figure 2. *ABCD* is a rectangle with *BC* = 120 cm and *DC* = 90 cm. The shape *EFG* is an isosceles triangle with *EG* = 60 cm and height 60 cm. The mid-point of *AD* and the mid-point of *EG* coincide.

(a) Find the distance of the centre of mass of the sign from the side *AD*.

(5)

The sign is freely suspended from *A* and hangs at rest.

(b) Find the size of the angle between *AB* and the vertical.

(4)

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Question 5 continued

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7. [*The centre of mass of a semi-circular lamina of radius r is* $\frac{4r}{3\pi}$ *from the centre*]

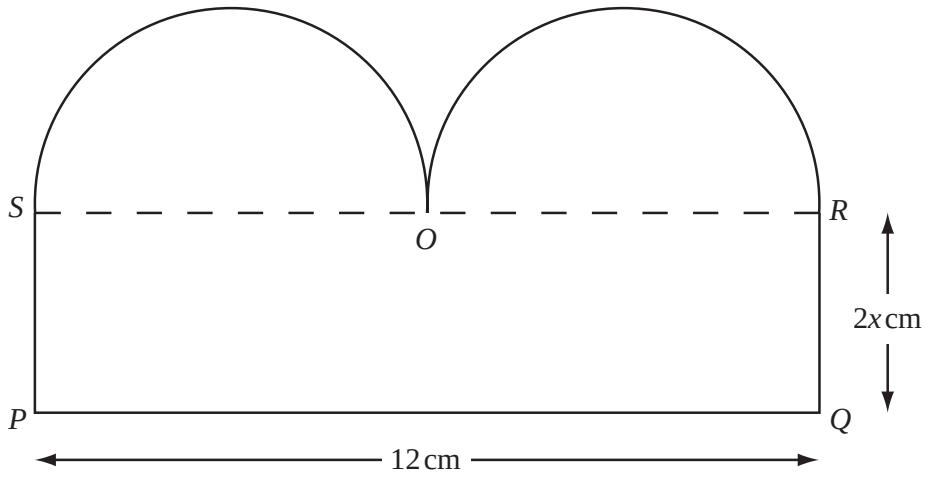

Figure 3

A template T consists of a uniform plane lamina $PQROS$, as shown in Figure 3. The lamina is bounded by two semicircles, with diameters SO and OR, and by the sides SP, PQ and QR of the rectangle $PQRS$. The point O is the mid-point of SR, $PQ = 12$ cm and $QR = 2x$ cm.

(a) Show that the centre of mass of T is a distance $\frac{4|2x^2 - 3|}{8x + 3\pi}$ cm from SR.

(7)

The template T is freely suspended from the point P and hangs in equilibrium.

Given that $x = 2$ and that θ is the angle that PQ makes with the horizontal,

(b) show that $\tan\theta = \frac{48 + 9\pi}{22 + 6\pi}$.

(4)

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Question 7 continued

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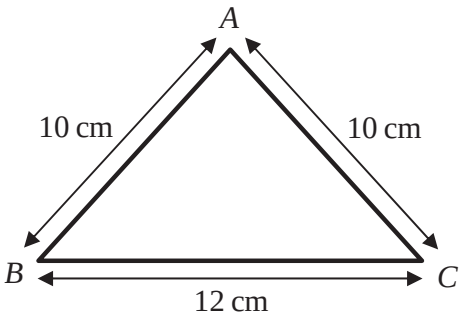

Figure 1

A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle ABC, where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in Figure 1.

(a) Find the distance of the centre of mass of the frame from BC.

(5)

The frame has total mass M. A particle of mass M is attached to the frame at the mid-point of BC. The frame is then freely suspended from B and hangs in equilibrium.

(b) Find the size of the angle between BC and the vertical.

(4)

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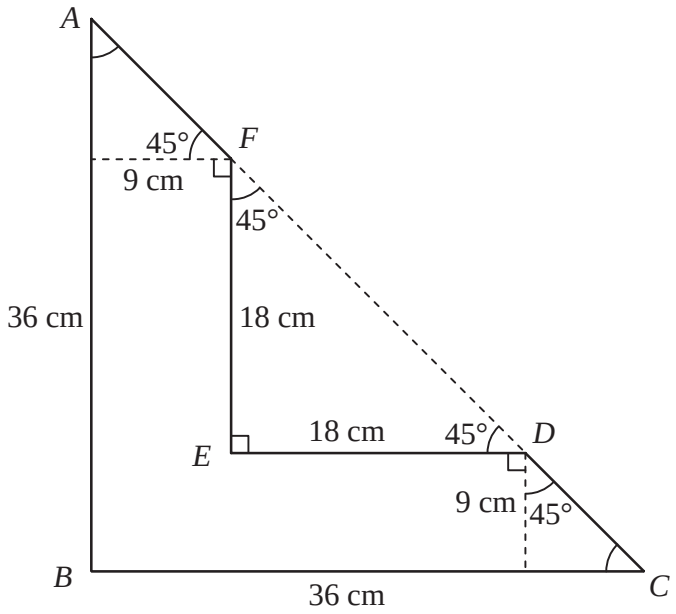

Figure 2

The uniform L-shaped lamina *ABCDEF*, shown in Figure 2, has sides *AB* and *FE* parallel, and sides *BC* and *ED* parallel. The pairs of parallel sides are 9 cm apart. The points *A*, *F*, *D* and *C* lie on a straight line.

AB = *BC* = 36 cm, *FE* = *ED* = 18 cm. $\angle ABC = \angle FED = 90°$, and $\angle BCD = \angle EDF = \angle EFD = \angle BAC = 45°$.

(a) Find the distance of the centre of mass of the lamina from

(i) side *AB*,

(ii) side *BC*.

(7)

The lamina is freely suspended from *A* and hangs in equilibrium.

(b) Find, to the nearest degree, the size of the angle between *AB* and the vertical.

(3)

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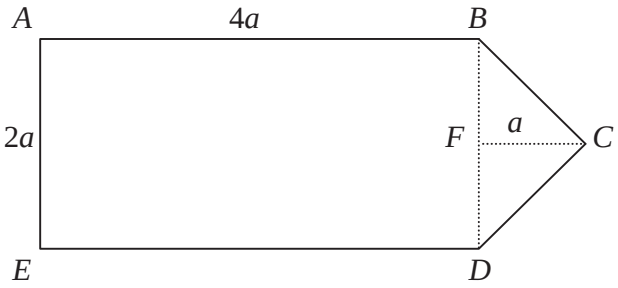

Figure 1

Figure 1 shows a uniform lamina $ABCDE$ such that $ABDE$ is a rectangle, $BC = CD$, $AB = 4a$ and $AE = 2a$. The point F is the midpoint of BD and $FC = a$.

(a) Find, in terms of a, the distance of the centre of mass of the lamina from AE.

(4)

The lamina is freely suspended from A and hangs in equilibrium.

(b) Find the angle between AB and the downward vertical.

(3)

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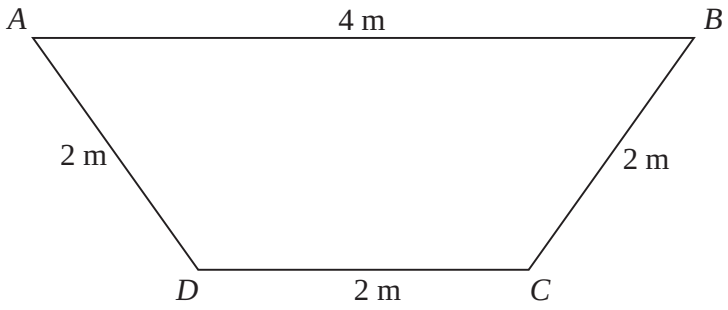

Figure 1

The trapezium $ABCD$ is a uniform lamina with $AB = 4$ m and $BC = CD = DA = 2$ m, as shown in Figure 1.

(a) Show that the centre of mass of the lamina is $\frac{4\sqrt{3}}{9}$ m from AB.

(5)

The lamina is freely suspended from D and hangs in equilibrium.

(b) Find the angle between DC and the vertical through D.

(5)

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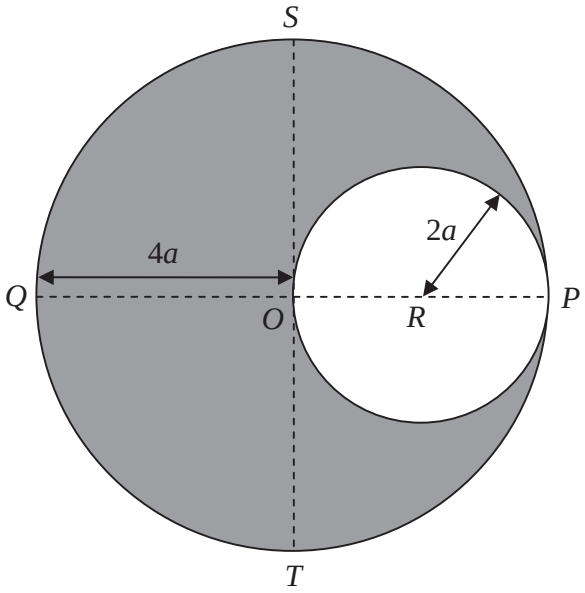

Figure 2

A uniform circular disc has centre O and radius $4a$. The lines PQ and ST are perpendicular diameters of the disc. A circular hole of radius $2a$ is made in the disc, with the centre of the hole at the point R on OP where $OR = 2a$, to form the lamina L, shown shaded in Figure 2.

(a) Show that the distance of the centre of mass of L from P is $\frac{14a}{3}$.

(4)

The mass of L is m and a particle of mass km is now fixed to L at the point P. The system is now suspended from the point S and hangs freely in equilibrium. The diameter ST makes an angle α with the downward vertical through S, where $\tan\alpha = \frac{5}{6}$.

(b) Find the value of k.

(5)

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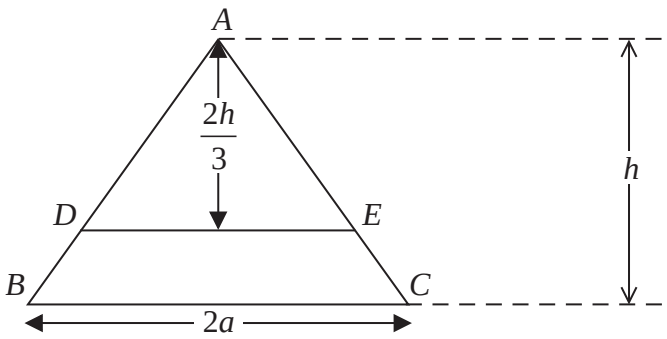

Figure 2

A uniform triangular lamina ABC of mass M is such that $AB = AC$, $BC = 2a$ and the distance of A from BC is h. A line, parallel to BC and at a distance $\frac{2h}{3}$ from A, cuts AB at D and cuts AC at E, as shown in Figure 2.

It is given that the mass of the trapezium $BCED$ is $\frac{5M}{9}$.

(a) Show that the centre of mass of the trapezium $BCED$ is $\frac{7h}{45}$ from BC.

(5)

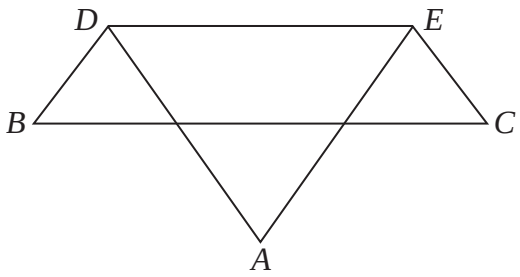

Figure 3

The portion ADE of the lamina is folded through 180° about DE to form the folded lamina shown in Figure 3.

(b) Find the distance of the centre of mass of the folded lamina from BC.

(4)

The folded lamina is freely suspended from D and hangs in equilibrium. The angle between DE and the downward vertical is α.

(c) Find $\tan \alpha$ in terms of a and h.

(4)

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Question 6 continued

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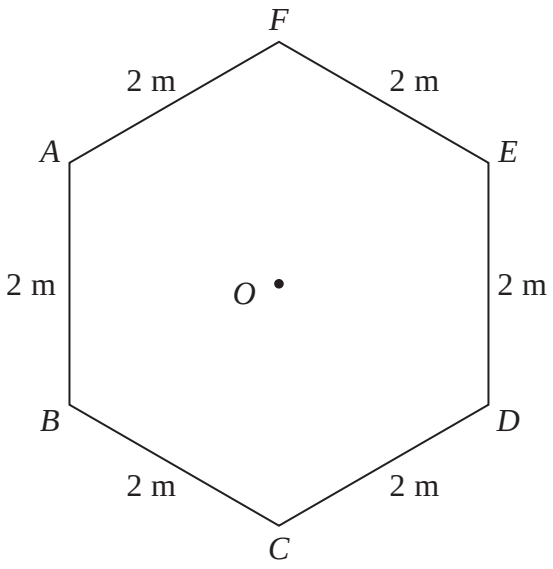

Figure 1

The uniform lamina *ABCDEF* is a regular hexagon with centre *O* and sides of length 2 m, as shown in Figure 1.

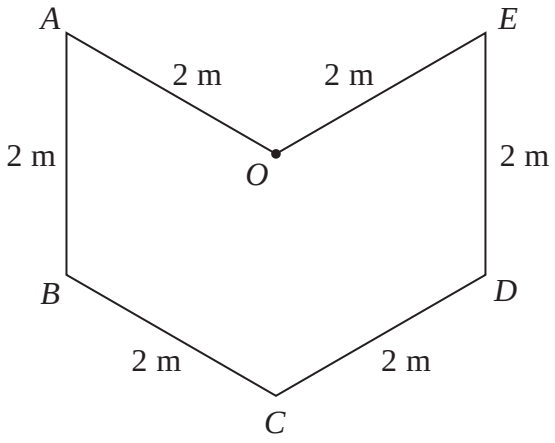

Figure 2

The triangles *OAF* and *OEF* are removed to form the uniform lamina *OABCDE*, shown in Figure 2.

(a) Find the distance of the centre of mass of *OABCDE* from *O*.

(5)

The lamina *OABCDE* is freely suspended from *E* and hangs in equilibrium.

(b) Find the size of the angle between *EO* and the downward vertical.

(6)

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Question 4 continued

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